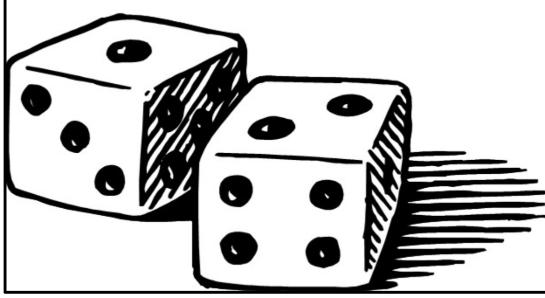
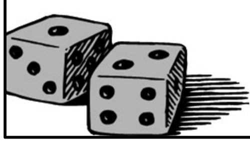
Probability

Algebra 2

Chapter 8



- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.



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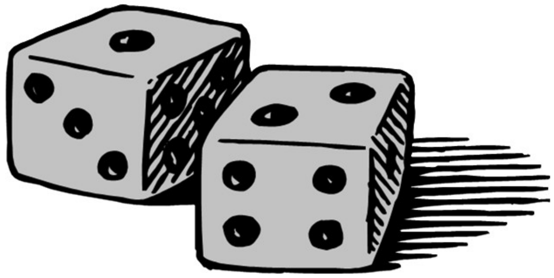


After this lesson…

- I can list the possible outcomes in a sample space.
- I can find theoretical probabilities.
- I can find experimental probabilities.

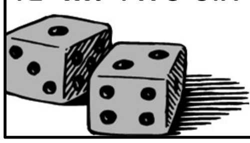
8.1 SAMPLE SPACES AND PROBABILITY

8.1 Sample Spaces and Probability

- **Work with a partner.**
- **a.** Describe the set of all possible outcomes for each experiment.
- **i.** Three coins are flipped.

- **ii.** One six-sided die is rolled.

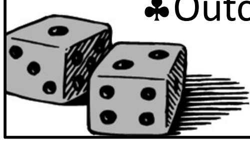
- **iii.** Two six-sided dice are rolled.



8.1 Sample Spaces and Probability

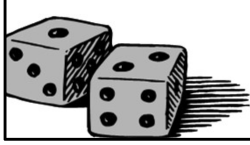
- Sample Spaces
  - The set of all possible outcomes in a probability experiment

  - Example
    - Probability Experiment: Flipping a Coin
    - Sample Space: H, T
    - Event (wanted outcome): H
    - Outcome (what happened): T



8.1 Sample Spaces and Probability

- 408#2 Find the number of possible outcomes and then list all the possible outcomes.
  You flip 4 coins



Number of outcomes: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

Make a tree diagram

Coin1: H T

Coin2: H T H T

Coin3: H T H T H T H T

Coin4: H T H T H T H T H T H T H T H T

HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT

8.1 Sample Spaces and Probability

- 408#1 Find the number of possible outcomes and then list all the possible outcomes. You flips a coin and draw a marble at random from a bag with 2 purple marbles and 1 white marble.

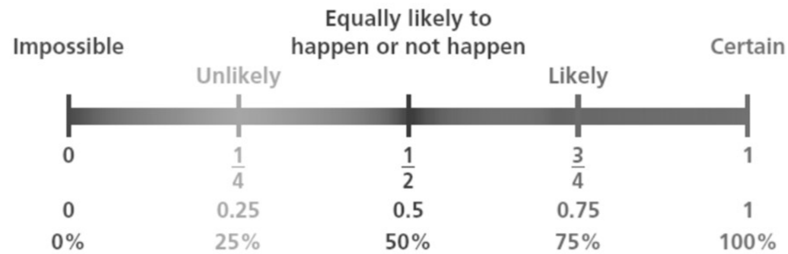
Number of outcomes: 2·3 = 6
Make a tree diagram

Coin:		H			T	
Marble:	P	P	W	P	P	W

HP, HP, HW, TP, TP, TW

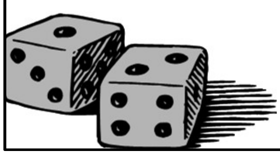
8.1 Sample Spaces and Probability

- Probability
  - A number between 0 and 1 to indicate how likely something is to happen
  - 0 = cannot happen
  - 1 = always happens



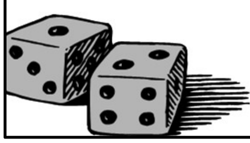
- Theoretical Probability

$$P(A) = \frac{\text{Number of ways A happens}}{\text{Total number of possible outcomes}}$$



8.1 Sample Spaces and Probability

- You flip a coin four times. What is the probability that the coins shows heads exactly three times?
- 408#5 A game show airs 5 days a week. Each day a prize is randomly placed behind one of two doors. What is the probability that exactly two contestant guess the correct door during a week?



Make a tree diagram
Coin1: H T
Coin2: H T H T
Coin3: H T H T H T H T
Coin4: H T H T H T H T H T H T H T H T

HHHH, **HHHT**, **HHTH**, HHTT, **HTHH**, HTHT, HTTH, HTTT, **THHH**, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT

$$P(3H) = \frac{4}{16} = \frac{1}{4} = 25\%$$

Sample Space:
Day1: W L
Day2: W L W
L
Day3: W L W L W L W
L
Day4: W L W L W L W L W L W L W
L W L
Day5: W L W L W L W L W L W L W L W L W L W L W L W L W L W L W L W L

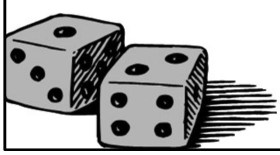
W L W L W L

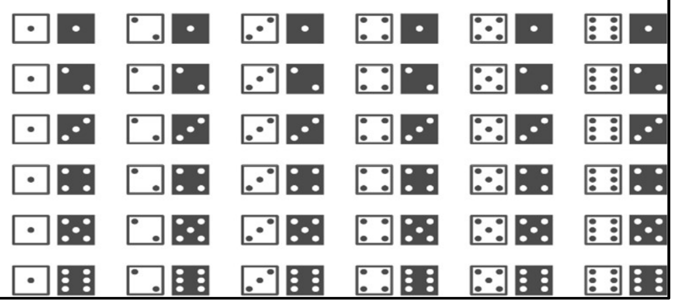
WWWWW, WWWWL, WWWLW, WWWLL, WWLWW, WWLWL, WWLLW, **WWLLL**, WLWWW, WLWWL, WLWLW, **WLWLL**, WLLWW, **WLLWL**, **WLLLW**, WLLLL
LWWWW, LWWWL, LWWLW, **LWWLL**, LWLWW, **LWLWL**, **LWLLW**, LWLLL, LLWWW, **LLWWL**, **LLWLW**, LLWLL, **LLLWW**, LLLWL, LLLLW, LLLLL

$$P(2W) = \frac{10}{32} = \frac{5}{16} = 31.25\%$$

8.1 Sample Spaces and Probability

- Two D6 are rolled. What is the probability of rolling a sum that is *not* 2?
- The sum is less than or equal to 10?
- Try #7 (a) *not* 4 (b) greater than 5





D6 means six-sided-dice

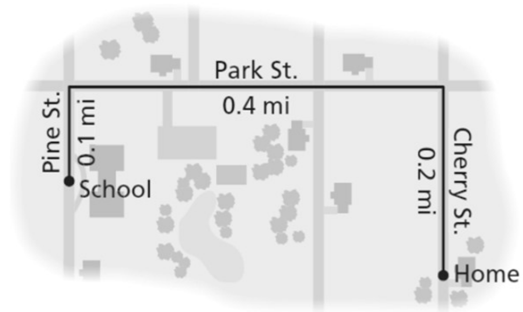
$$P(\sim 2) = \frac{35}{36} \approx 0.972$$

$$P(n \leq 10) = \frac{33}{36} = \frac{11}{12} \approx 0.917$$

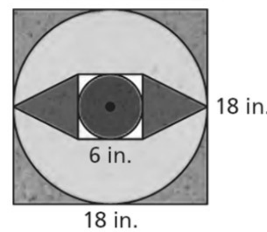
#7 $P(\sim 4) = \frac{33}{36} = \frac{11}{12} \approx 92\%; P(> 5) = \frac{26}{36} = \frac{13}{18} \approx 72\%$

8.1 Sample Spaces and Probability

- 408#12 A student loses his earbuds while walking home form school. The earbuds are equally likely to be at any point along the path shown. What is the probability that the earbuds are on Cherry Street?



- Try #11 Probability of Yellow



$$P(Cherry\ St.) = \frac{Cherry\ Str}{Whole\ path} = \frac{0.2\ mi}{0.1\ mi + 0.4\ mi + 0.2\ mi} = \frac{0.2}{0.7} = \frac{2}{7} = 28.6\%$$

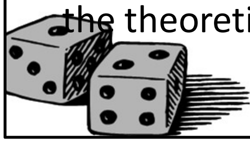
$$P(Y) = \frac{Yellow\ area}{total\ area} = \frac{\pi 9^2 - 6^2 - 2\left(\frac{1}{2} 6 \cdot 6\right)}{18^2} = \frac{182.5}{324} \approx 56.3\%$$

8.1 Sample Spaces and Probability

- Experimental Probability
  - Probability based on the results of an experiment
- Each section of the spinner shown has the same area. The spinner is spun 50 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?

Spinner Results				
red	green	blue	yellow	purple
5	20	3	10	12

Try 409#13

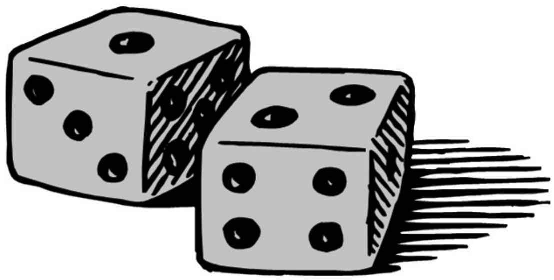


408 #1, 2, 3, 5, 7, 8, 9, 11, 12, 13, 15, 17, 19, 23, 25, 29, 33, 35, 37, 39

Theoretical Probability: 1/5
Experimental Probability:
Red: 5/50=1/10
Green: 20/50=2/5
Blue: 3/50
Yellow: 10/50=1/5
Purple: 12/50=6/25

Yellow is the same theoretical and experimental

#13 Theoretical Probability for each color = 1/5=20%
Experimental
White: $\frac{5}{30} = \frac{1}{6} \approx 16.7\%$
Black: $\frac{6}{30} = \frac{1}{5} = 20\%$
Red: $\frac{8}{30} = \frac{4}{15} \approx 26.7\%$
Green: $\frac{2}{30} = \frac{1}{15} \approx 6.7\%$
Blue: $\frac{9}{30} = \frac{3}{10} = 30\%$
Blue is higher

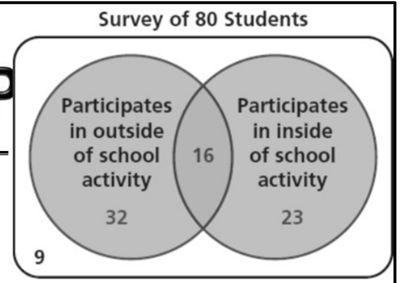


After this lesson…

- I can make two-way tables.
- I can find and interpret relative frequencies and conditional relative frequencies.
- I can use conditional relative frequencies to find probabilities.

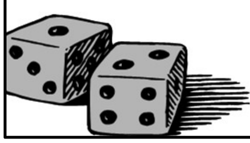
8.2 TWO-WAY TABLES AND PROBABILITY

8.2 Two-Way Tables and P



- **Work with a partner.** A survey of 80 students at a high school asks whether they participate in outside of school activities and whether they participate in inside of school activities. The results are shown in the Venn diagram.

- **a.** Show how you can represent the data in the Venn diagram using a single table.

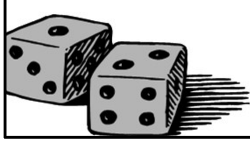


8.2 Two-Way Tables and Probability

- Two-Way Table
  - Displays data from one source that belongs to two different categories
  - Entries are joint frequencies
  - Totals are marginal frequencies

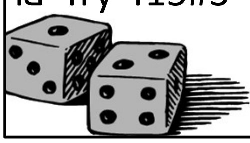
		Attendance	
		Attending	Not Attending
Class	Freshman	25	44
	Sophomore	80	32

joint frequency



8.2 Two-Way Tables and Probability

- There are 16 juniors and 24 seniors on a debate team. Of those, 7 juniors and 19 seniors qualify for a state debate competition. Organize this information in a two-way table. Then find and interpret the marginal frequencies.
- Try 415#3



		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.			
	Sr.			
	Total			

7	9	16
19	5	24
26	14	40

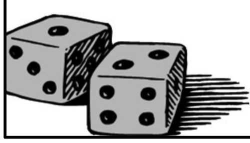
40 students are on the debate team, 26 students qualified, 14 students did not qualify, 16 juniors qualified, and 24 seniors qualified.

#3	M	F	Total
Y	132	151	283
N	39	29	68
Total	171	180	351

351 people were surveyed, 171 males were surveyed, 180 females were surveyed, 283 people said yes, 68 people said no.

8.2 Two-Way Tables and Probability

- Relative Frequencies
  - Joint Relative Frequency
    - Ratio of joint frequency to total values
  - Marginal Relative Frequency
    - Sum of joint relative frequencies in a row or column



8.2 Two-Way Tables and Probability

- Make a table showing the relative frequencies.

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.	7	9	16
	Sr.	19	5	24
	Total	26	14	40

Try 415#5

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.			
	Sr.			
	Total			

$\frac{7}{40} = 0.175$ $\frac{9}{40} = 0.225$ $\frac{16}{40} = 0.4$

$\frac{19}{40} = 0.475$ $\frac{5}{40} = 0.125$ $\frac{24}{40} = 0.6$

$\frac{26}{40} = 0.65$ $\frac{14}{40} = 0.35$ $\frac{40}{40} = 1$

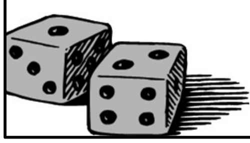
The joint relative frequency 0.475 means that 47.5% of the students on the debate team are seniors who qualify for the state debate competition. So, the probability that a randomly selected student from the debate team is a senior that qualified for the state debate competition is 47.5%.

The marginal relative frequency 0.4 means that 40% of the students on the debate team are juniors. So, the probability that a randomly selected student from the debate team is a junior is 40%.

#5	M	F	Total
Y	0.376	0.430	0.806
N	0.111	0.083	0.194
Total	0.487	0.513	1

8.2 Two-Way Tables and Probability

- Conditional Relative Frequencies
  - Ratio of a joint relative frequency to the marginal relative frequency
  - Can be done for row totals or column totals



8.2 Two-Way Tables and Probability

- Make a two-way table that shows the conditional relative frequencies based on (a) the row totals

		State Competition		
		Qualified	Not Qualified	Total
Class	Jr.	7	9	16
	Sr.	19	5	24
	Total	26	14	40

Try 415#7

415 #1, 2, 3, 5, 7, 9, 11, 13, 14, 15, 19, 21, 25, 27, 29

		State Competition	
		Qualified	Not Qualified
Class	Jr.		
	Sr.		

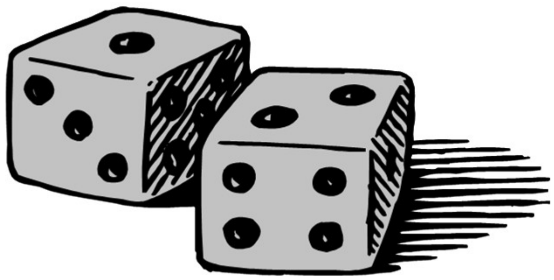
$\frac{7}{16} = 0.438$ $\frac{9}{16} = 0.563$

$\frac{19}{24} = 0.792$ $\frac{5}{24} = 0.208$

The conditional relative frequency 0.792 means that about 79.2% of the seniors on the debate team qualified for the state debate competition. So, the probability that a randomly selected senior from the debate team qualified for the state debate competition is about 79.2%.

#7a	Y	N
Y	0.534	0.483
N	0.466	0.517

#7b	Y	N
Y	0.481	0.519
N	0.431	0.569



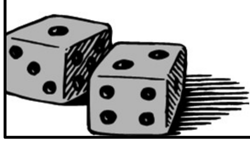
After this lesson…

- I can explain the meaning of conditional probability.
- I can fi nd conditional probabilities.
- I can make decisions using probabilities.

8.3 CONDITIONAL PROBABILITY

8.3 Conditional Probability

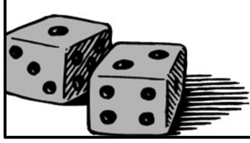
- **Work with a partner.** Six pieces of paper, numbered 1 through 6, are placed in a bag. You draw two pieces of paper one at a time without replacing the first.
- **b.** What is the probability that you draw two odd numbers?
- **c.** When the first number you draw is odd, what is the probability that the second number you draw is also odd? Explain.
- **d.** Compare and contrast the questions in parts (b) and (c).



b. $\frac{6}{30} = \frac{1}{5} = 20\%$ (1-3, 1-5, 3-1, 3-5, 5-1, 5-3) total 6·5=30
c. $\frac{2}{5} = 40\%$ (two or five remaining numbers are odd)

8.3 Conditional Probability

- Conditional Probability
  - Probability that B occurs given that A has already occurred
  - $P(B|A)$



8.3 Conditional Probability

- A family has three rabbits and two guinea pig. They randomly select a pet to get brushed and then randomly select a different pet to get a treat. Find the probability that they select a rabbit to get a treat given that they selected the guinea pig to get brushed.

- Try 422#1

Sample Space:

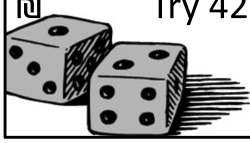
R1, R2	R2, R1	R3, R1	**G1, R1**	**G2, R1**
R1, R3	R2, R3	R3, R2	**G1, R2**	**G2, R2**
R1, G1	R2, G1	R3, G1	**G1, R3**	**G2, R3**
R1, G2	R2, G2	R3, G2	**G1, G2**	**G2, G1**

$$P(R|G) = \frac{6}{8} = \frac{3}{4} = 0.75$$

#1 $\frac{2\ veg}{4\ options} = 50\%$

8.3 Conditional Probability

- A quality-control inspector checks for defective parts. The two-way table shows the results. Find each probability.
- *P*(pass | defective)
- *P*(pass | non-defective)
- Try 422#3



		Result	
		Pass	Fail
Part Type	Defective	5	24
	Non-defective	208	9

$$P(pass|defective) = \frac{5}{5+24} = \frac{5}{29}$$

$$P(pass|nondefective) = \frac{208}{208+9} = \frac{208}{217}$$

#3 $P(pass \mid Test\ A) = \frac{49}{49+7} 87.5 = \%;\ P(Test\ C \mid fail) = \frac{12}{7+6+12} = 48\%$

8.3 Conditional Probability

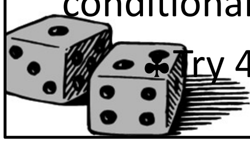
₪ Conditional Probability Formula

♥ $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

		Result	
		Pass	Fail
Part Type	Defective	5	24
	Non-defective	208	9

₪ Find *P*(pass | non-defective) using the formula for conditional probability.

♣ Try 422#5



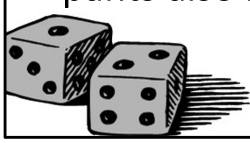
$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(pass|nondefective) = \frac{\frac{208}{246}}{\frac{208+9}{246}} = \frac{0.846}{0.882} = 0.959$$

#5 $P(pass \mid Test\ A) = \frac{P(pass\ and\ Test\ A)}{P(Test\ A)} = \frac{\frac{49}{154}}{\frac{56}{154}} = 87.5\%$

8.3 Conditional Probability

- At a clothing store, 75% of customers buy a pair of pants, 24% of customers buy a belt, and 20% of customers buy a pair of pants and a belt.
- What is the probability that a customer who buys a pair of pants also buys a belt?
- What is the probability that a customer who buys a belt also buys a pair of pants?
- Try 422#9



$$P(belt|pants) = \frac{P(belt\ and\ pants)}{P(pants)} = \frac{0.20}{0.75} = 0.267$$

$$P(pants|belt) = \frac{P(pants\ and\ belt)}{P(belt)} = \frac{0.20}{0.24} = 0.833$$

#9 $P(Dance \mid Game) = \frac{P(dance\ and\ game)}{P(game)} = \frac{0.23}{0.43} = 53.5\%$; $P(Game \mid Dance) = \frac{P(game\ and\ dance)}{P(dance)} = \frac{0.23}{0.48} = 47.9\%$

8.3 Conditional Probability

- An airline company strives to not lose luggage. A manager is evaluating three flights in order to determine which flight loses luggage the least often. At the end of each day, the manager records whether or not luggage was lost on the flights that day. The table shows the results. Which flight loses luggage the least often?
- Try 423#11

Flight	Lost Luggage	No Lost Luggage
A	卌 卌	III
B	卌 IIII	卌
C	卌 卌 II	I

422 #1, 3, 5, 7, 9, 10, 11, 13, 15, 16, 17, 18, 19, 21, 25, 29, 31, 33, 35, 37

Make two-way table with shows the joint and marginal relative frequencies (40 marks)

	Lost	Not Lost	Total
Flight A	0.250	0.075	0.325
Flight B	0.225	0.125	0.350
Flight C	0.300	0.025	0.325
Total	0.775	0.225	1

$$P(\sim lost|A) = \frac{P(A\ and\ \sim lost)}{P(A)} = \frac{0.075}{0.325} = 0.231$$
$$P(\sim lost|B) = \frac{P(B\ and\ \sim lost)}{P(B)} = \frac{0.125}{0.350} = 0.357$$
$$P(\sim lost|C) = \frac{P(C\ and\ \sim lost)}{P(C)} = \frac{0.025}{0.325} = 0.077$$

Flight B has highest probability of not losing luggage.

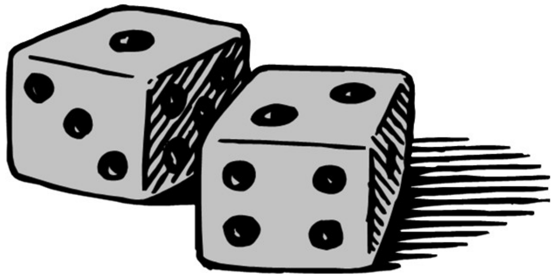
#11

$$P(late|A) = \frac{P(late\ and\ A)}{P(A)} = \frac{\frac{4}{41}}{\frac{11}{41}} = \frac{4}{11} = 36.4\%$$

$$P(late|B) = \frac{P(late\ and\ B)}{P(B)} = \frac{3}{14} = 21.4\%$$

$$P(late|C) = \frac{P(late\ and\ C)}{P(C)} = \frac{4}{16} = 25\%$$

Choose route B



After this lesson…

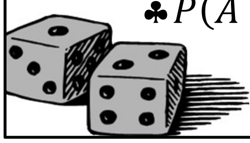
- I can explain how independent events and dependent events are different.
- I can determine whether events are independent.
- I can find probabilities of independent and dependent events.

8.4 INDEPENDENT AND DEPENDENT EVENTS

8.4 Independent and Dependent Events

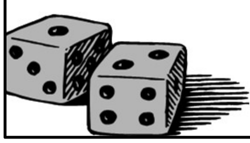
- 2 events → 2 outcomes
- Independent Events
  - One event does not affect the other event
  - $P(A\ and\ B) = P(A) \cdot P(B)$
  - $P(A|B) = P(A)$ and $P(B|A) = P(B)$

- Dependent Events
  - One event does affect the other event
    - $P(A\ and\ B) = P(A) \cdot P(B|A)$



8.4 Independent and Dependent Events

- A bag contains six pieces of paper, numbered 1 through 6. You randomly select a piece of paper, replace it, and then randomly select another piece of paper. Use a sample space to determine whether randomly selecting a 5 first and randomly selecting an odd number second are independent events.



Independent because there are 6 choices both times you draw

8.4 Independent and Dependent Events

- A bag contains six pieces of paper, numbered 1 through 6. You randomly select a piece of paper, set it aside, and then randomly select another piece of paper. Use a sample space to determine whether randomly selecting an even number first and randomly selecting a 4 second are independent events.

- Try 430#1, 5

Dependent because there are 6 choices the first time, but only 5 choices the second time. One of the choices has been removed

#1: Independent – The spinner has same probability with each spin
#5: Dependent – The number of choices changes after the first draw

8.4 Independent and Dependent Events

- A store surveys customers of different ages. The survey asks whether they want to see the store expand its toy department. The results, given as joint relative frequencies, are shown in the two-way table. Determine whether wanting to see the store expand and being less than 10 years old are independent events.
- Try 430#11

		Age (in years)		
		< 10	10–20	> 20
Response	Yes	0.27	0.06	0.23
	No	0.09	0.17	0.18

not independent;

$$P(< 10) = 0.27 + 0.09 = 0.36$$

$$P(< 10 \mid Yes) = \frac{P(< 10\ and\ Yes)}{P(Yes)} = \frac{0.27}{0.27 + 0.06 + 0.23} = 0.48$$

P(less than 10 years old) ≠ *P*(less than 10 years old | yes)

#11: P(Yes) = 0.75; P(Yes | Saratoga) = $\frac{P(yes\ and\ Saratoga)}{P(Saratoga)} = \frac{0.289}{0.289+0.095} = 0.75$

Independent because P(Yes) = P(Yes | Saratoga)

8.4 Independent and Dependent Events

- Find the probability that you get an even number on your first spin and a number less than 3 on your second spin.

- Try 431#13

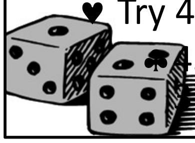
Independent

$$P(Even\ and < 3) = P(even) \cdot P(< 3)$$
$$= \frac{4}{8} \cdot \frac{2}{8}$$
$$= \frac{1}{2} \cdot \frac{1}{4}$$
$$= \frac{1}{8} = 0.125$$

#13: P(>500 and BR) = P(>500)P(BR) = $\frac{8}{24} \cdot \frac{2}{24} = \frac{1}{36} \approx 0.028 = 2.8\%$

8.4 Independent and Dependent Events

- Nine women and six men are on a committee. One person is randomly selected to be the chairperson and a different person is randomly selected to be the treasurer. Find the probability that both events *A* and *B* will occur.
- **Event *A***: The chairperson is a man.
- **Event *B***: The treasurer is a woman.

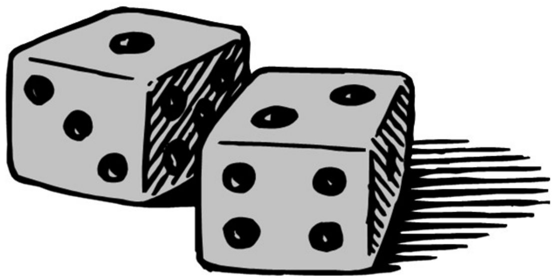


- Try 431#15
- 430 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 41

Dependent

$$P(M \text{ and } W) = P(M) \cdot P(W|M)$$
$$= \frac{6}{15} \cdot \frac{9}{14} = 0.257$$

#15: $P(concert\ and\ concert) = P(concert)P(concert \mid concert) = \frac{8}{20} \cdot \frac{7}{19} = \frac{14}{95} \approx 0.147 = 14.7\%$



After this lesson…

• I can explain how disjoint events and overlapping events are different.

• I can find probabilities of disjoint events.

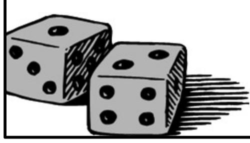
• I can find probabilities of overlapping events.

• I can solve real-life problems using more than one probability rule.

8.5 PROBABILITY OF DISJOINT AND OVERLAPPING EVENTS

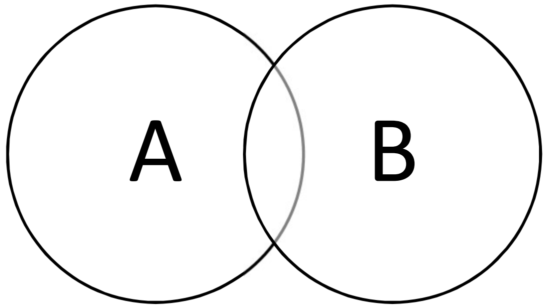
8.5 Probability of Disjoint and Overlapping Events

- Compound Event
  - 1 event with 2 acceptable outcomes
- There may be some intersections where one condition satisfies both events so the events are overlapping
- If there is no intersection, then they are disjoint or mutually exclusive



8.5 Probability of Disjoint and Overlapping Events

- $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

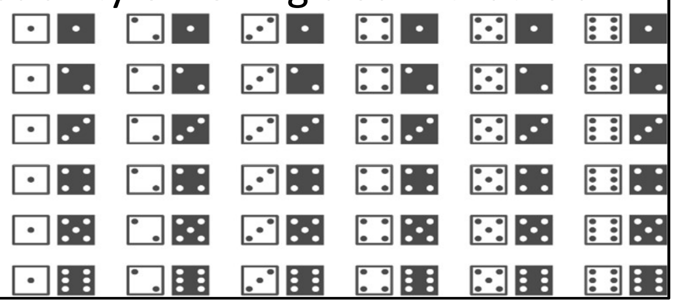


- If they are disjoint or mutually exclusive
  - $P(A\ and\ B) = 0$

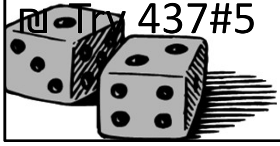
The overlap (A and B) is counted twice (once with A and once with B) so it is subtracted once.

8.5 Probability of Disjoint and Overlapping Events

- One D6 is rolled. What is the probability of rolling a multiple of 3 or 5?
- Two D6 are rolled. What is the probability of rolling a sum that is a multiple of 2 or 3?



- Try 437#5



D6 means six-sided-dice

P(mult 3 or 5) = P(mult 3) + P(mult 5) – P(mult 3 and 5)
P(mult 3 or 5) = 2/6 + 1/6 – 0 = 3/6 = ½

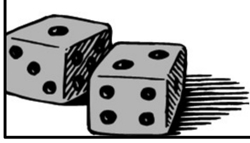
P(mult 2 or 3) = P(mult 2) + P(mult 3) – P(mult 2 and 3)
P(mult 2 or 3) = 3/6 + 2/6 – 1/6 = 4/6 = 2/3

#5: $P(R\ or\ odd) = P(R) + P(odd) - P(R\ and\ odd) = \frac{4}{12} + \frac{6}{12} - \frac{2}{12} = \frac{8}{12} = \frac{2}{3} \approx 0.667 = 66.7\%$

8.5 Probability of Disjoint and Overlapping Events

₪ A bag contains twenty cards, numbered 1 through 20. A card is randomly selected. What is the probability that the number is a multiple of 3 *or* a multiple of 4?

₪ Try #13

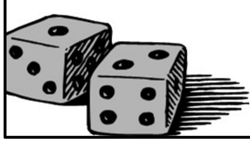


$$P(\times 3\ or \times 4) = P(\times 3) + P(\times 4) - P(\times 3\ and \times 4)$$
$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20}$$
$$= \frac{1}{2} = 0.5$$

#13: $P(D\ or\ I) = P(D) + P(I) - P(D\ and\ I) \rightarrow \frac{34}{40} = \frac{18}{40} + \frac{20}{40} - P(D\ and\ I) \rightarrow -\frac{4}{40} = -P(D\ and\ I) \rightarrow P(D\ and\ I) = \frac{1}{10} = 0.1 = 10\%$

8.5 Probability of Disjoint and Overlapping Events

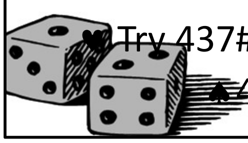
- Out of 45 customers at a breakfast café, 42 customers bought either coffee or orange juice. There were 30 customers who bought orange juice and 40 customers who bought coffee. What is the probability that a randomly selected customer bought both coffee and orange juice?



$$P(C\ or\ O) = P(C) + P(O) - P(C\ and\ O)$$
$$\frac{42}{45} = \frac{40}{45} + \frac{30}{45} - P(C\ and\ O)$$
$$P(C\ and\ O) = \frac{28}{45} = 0.622$$

8.5 Probability of Disjoint and Overlapping Events

- A medical association estimates that 10.9% of the people in the United States have a thyroid disorder. A medical lab develops a simple diagnostic test for the disorder that is 96% accurate for people who have the disorder and 99% accurate for people who do not have it. The medical lab gives the test to a randomly selected person. What is the probability that the diagnosis is correct?



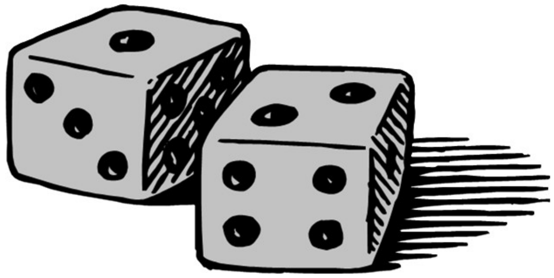
- Try 437#15
  - 437 #1-30 odd

Tree diagram

```
                    people
                  /        \
           0.109             0.891
           with              without
      /          \          /        \
0.96          0.04      0.99          0.01
Accurate      Not       Accurate      Not
```

$$P(accurate) = P((with\ and\ accurate)\ or\ (without\ and\ accurate))$$
$$= P(with\ and\ accurate) + P(without\ and\ accurate)$$
$$= P(with)P(accurate|with) + P(without)P(accurate|without)$$
$$= 0.109 \cdot 0.96 + 0.891 \cdot 0.99$$
$$= 0.987$$

#15: $P(buy) = P\big((M\ and\ buy)\ or\ (F\ and\ buy)\big) = P(M\ and\ buy) + P(F\ and\ buy) = P(M)P(buy|M) + P(F)P(buy|F) = (0.47)(0.40) + (0.53)(0.54) = 0.4742 = 47.42\%$



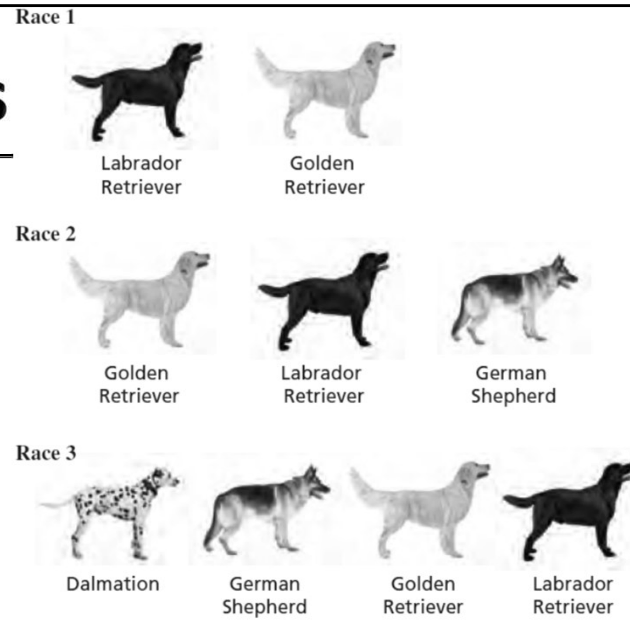
After this lesson…

- I can explain the difference between permutations and combinations.
- I can find numbers of permutations and combinations.
- I can find probabilities using permutations and combinations.

8.6A PERMUTATIONS AND COMBINATIONS

8.6A Permutations

- **Work with a partner.**
- **a.** A fair conducts three obstacle course races. In how many different orders can the dogs finish in each race? Justify your answers.
- **b.** For each race in part (a), in how many different ways can the dogs finish first and second? Justify your answers.
- **c.** For each race in part (a), how many different pairs of dogs can you form?
- **d.** Explain why your answers in part (c) are different from your answers in part (b)



a. Race 1: 2 orders; Race 2: 6 orders; Race 3: 24 orders
b. Race 1: 2 orders; Race 2: 6 orders; Race 3: 12 orders
c. Race 1: 1 pair; Race 2: 3 pairs; Race 3: 6 pairs
d. Parts a and b have order, part c does not have order

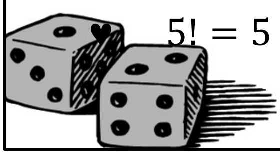
8.6A Permutations and Combinations

- Permutation
  - How many ways to order objects
    - A, B, C →
    - ABC, ACB, BAC, BCA, CAB, CBA → 6 ways

- Number of Permutations of *n* objects taken *r* at a time

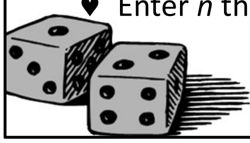
$$_nP_r = \frac{n!}{(n-r)!}$$

- Factorial (!) – that number times all whole numbers less than it
  - $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$



8.6A Permutations and Combinations

- Permutations on a Calculator
- TI
  - Enter value of *n*
  - Press MATH → PRB ↓ nPr
  - Enter value of *r*
- NumWorks
  - Press Toolbox button
  - Down to Probability
  - Down to Combinatorics
  - Permute(n,k)
  - Enter *n* then *r*



- 445#7
  Evaluate ${}_5P_2$
- Try 445#9
  Evaluate ${}_9P_1$

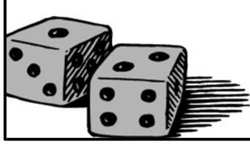
Use the calculator

$$ {}_5P_2 = 20 $$

$$ {}_9P_1 = 9 $$

8.6A Permutations and Combinations

- Consider the letters in the word PENCILS.
- In how many ways can you arrange all of the letters?
- In how many ways can you arrange 3 of the letters?
- Try 445#3 ROCK



Order = permutation

$$_7P_7 = 5040$$

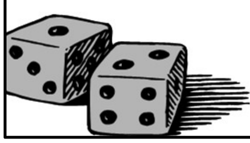
$$_7P_3 = 210$$

#3

$$_4P_4 = 24$$
$$_4P_2 = 12$$

8.6A Permutations and Combinations

- Eight people serve on a committee. In how many different ways can a chairperson, a recorder, and a treasurer be chosen from the committee members?
- Try 445#13
  Eleven students are competing in a graphic design contest. In how many different ways can the students finish first, second, and third?



Order = permutation

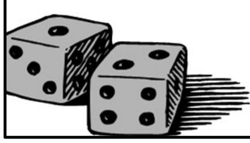
$${}_{8}P_{3} = 336$$

#13

$${}_{11}P_{3} = 990$$

8.6A Permutations and Combinations

- You and your friend are auditioning for a part in the school play. There are 15 people auditioning, and the order of their auditions is chosen at random. Find the probability that your audition is last and your friend's audition is second to last.
- Try 442#15
  You and your friend are 2 of 8 servers working a shift in a restaurant. At the beginning of the shift, the manager randomly assigns one section to each server. Find the probability that you are assigned Section 1 and your friend is assigned Section 2



$$P(last\ 2) = \frac{{}_2P_2}{{}_{15}P_2} = \frac{1}{210} \approx 0.00476$$

#15

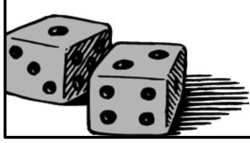
$$P(2\ sections) = \frac{{}_2P_2}{{}_8P_2} = \frac{1}{56} \approx 0.018$$

8.6A Permutations and Combinations

- Combination
  - Arranging of objects **without** order

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

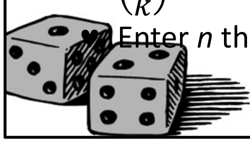
- Permutation have order
- Combination do not have order



Permutations had order, Combinations are without order

8.6A Permutations and Combinations

- Combinations on a Calculator
- TI
  - Enter value of *n*
  - Press MATH → PRB ↓ nCr
  - Enter value of *r*
- NumWorks
  - Press Toolbox button
  - Down to Probability
  - Down to Combinatorics
  - $\binom{n}{k}$
  - Enter *n* then *r*



- 445#21
  Evaluate ${}_5C_1$
- Try 445#23
  Evaluate ${}_9C_9$

#21

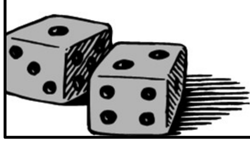
$${}_5C_1 = 5$$

#23

$${}_9C_9 = 1$$

8.6A Permutations and Combinations

- Count the possible combinations of 4 letters chosen from the list P, Q, R, S, T, U.
- Try 445#17
  A, B, C, D; *r* = 3



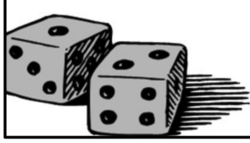
$_6C_4 = 15$

#17

$_4C_3 = 4$

8.6A Permutations and Combinations

- You are listening to music. You have time to listen to 3 songs from your playlist of 16 songs. How many combinations of 3 songs are possible?
- Try 445#27
  A team of 25 rowers attends a rowing tournament. Five rowers compete at a time. How many combinations of 5 rowers are possible?



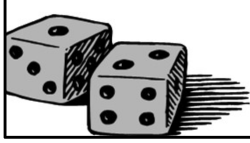
$${}_{16}C_3 = 560$$

#27

$${}_{25}C_5 = 53130$$

8.6A Permutations and Combinations

- Tell whether to use a permutation or combination, then answer the question.
- 446#33
  To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?
- Try 446#35
  Fifty-two athletes are competing in a bicycle race. In how many orders can the bicyclists finish first, second, and third?



#33: Combination because no order

$$_{10}C_8 = 45$$

#35: Permutation because order

$$_{52}P_3 = 132600$$

8.6A Permutations and Combinations

- An art teacher has selected 13 projects, including one of yours and one of your friend's, to put into a display case in the hallway. The projects are placed at random. There is room for 2 projects in the middle row of the case. What is the probability that your project and your friend's project are the 2 placed in the middle row?

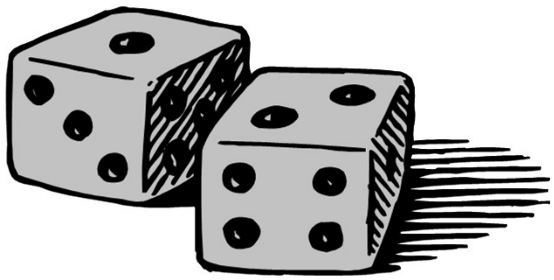
- Try 446#37
  You and your friend are in the studio audience on a game show. From an audience of 300 people, 2 people are randomly selected as contestants. What is the probability that your and your friend are chosen?

445 #1, 3, 5, 7, 9, 13, 14, 15, 16, 17, 19, 21, 23, 27, 28, 33, 34, 35, 37, 38

$$P(2\ middle) = \frac{{}_2C_2}{{}_{13}C_2} = \frac{1}{78} = 0.013$$

#37

$$P(2\ people) = \frac{{}_2C_2}{{}_{300}C_2} = \frac{1}{44850}$$



After this lesson…

- I can expand powers of binomials using the binomial theorem.
- I can find coefficients in a binomial expansion.

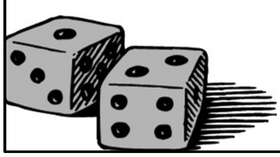
8.6B THE BINOMIAL THEOREM

8.6B Combinations and the Binomial Theorem

₪ Binomial Theorem

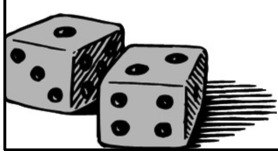
♥ $(a+b)^n = {}_nC_0 a^{n-0} b^0 + {}_nC_1 a^{n-1} b^1 + \cdots + {}_nC_r a^{n-r} b^r$

$$= \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$



8.6B Combinations and the Binomial Theorem

- 446#48
  Expand $(c-4)^5$



$${}_5C_0c^5(-4)^0 + {}_5C_1c^4(-4)^1 + {}_5C_2c^3(-4)^2 + {}_5C_3c^2(-4)^3 + {}_5C_4c^1(-4)^4 + {}_5C_5c^0(-4)^5$$

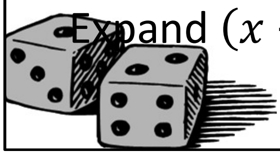
$$1c^5 1 + 5c^4(-4) + 10c^3 16 + 10c^2(-64) + 5c^1 256 + 1 \cdot 1 \cdot (-1024)$$

$$c^5 - 20c^4 + 160c^3 - 640c^2 + 1280c - 1024$$

8.6B Combinations and the Binomial Theorem

- 446#51
  Expand $(w^3 - 3)^4$

- Try 446#47
  Expand $(x + 2)^3$



$$ {}_4C_0(w^3)^4(-3)^0 + {}_4C_1(w^3)^3(-3)^1 + {}_4C_2(w^3)^2(-3)^2 + {}_4C_3(w^3)^1(-3)^3 + {}_4C_4(w^3)^0(-3)^4 $$

$$1 \cdot w^{12} \cdot 1 + 4 \cdot w^9(-3) + 6 \cdot w^6(9) + 4 \cdot w^3(-27) + 1 \cdot 1(81)$$

$$w^{12} - 12w^9 + 54w^6 - 108w^3 + 81$$

#47

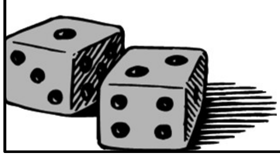
$$ {}_3C_0x^3 2^0 + {}_3C_1x^2 2^1 + {}_3C_2x^1 2^2 + {}_3C_3x^0 2^3 $$

$$1x^3 1 + 3x^2 2 + 3x4 + 1 \cdot 1 \cdot 8$$

$$x^3 + 6x^2 + 12x + 8$$

8.6B Combinations and the Binomial Theorem

- 446#56
  Find the coefficient of the x^4 term in $(x-3)^7$
- Try 446#55
  Find the coefficient of the x^5 term in $(x-2)^{10}$



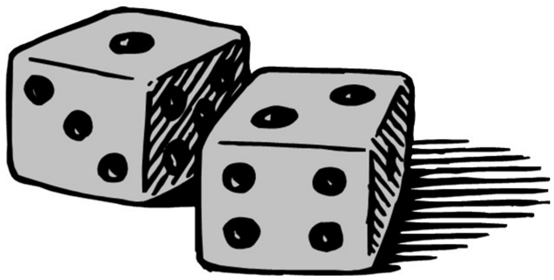
- 445 #47, 48, 49, 51, 53, 55, 56, 57, 58, 59, 67, 71, 83, 85, 87

#56

$$n = 7, n - r = 4 \rightarrow 7 - r = 4 \rightarrow r = 3$$
$$_7C_3 a^{7-3} b^3 = 35 \cdot (x)^4(-3)^3 \rightarrow -945x^4$$

#55

$$n = 10, n - r = 5 \rightarrow 10 - r = 5 \rightarrow r = 5$$
$$_{10}C_5 a^{10-5} b^5 = 252 \cdot x^5 \cdot (-2)^5 = -8064x^5$$



After this lesson…

- I can explain the meaning of a probability distribution.
- I can construct and interpret probability distributions.
- I can find probabilities using binomial distributions.

8.7 BINOMIAL DISTRIBUTIONS

8.7 Binomial Distributions

₪ Probability Distribution

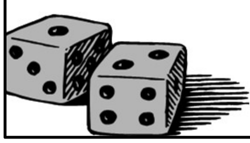
♥ Function that gives the probability of each of the possible outcomes in a probability experiment

♥ The sum of the probabilities = 1

₪ Construct a Probability Distribution

♥ Make a table of probabilities

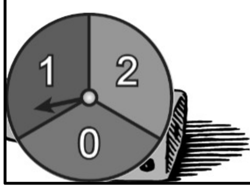
♥ Make a histogram



Probability Distribution for Rolling a Six-Sided Die						
x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

8.7 Binomial Distributions

- The spinner is divided into three equal parts. Let *x* be a random variable that represents the sum when the spinner is spun twice. Make a table and draw a histogram showing the probability distribution for *x*.



- Try 453#1

Sample Space

(0, 0)	(1, 0)	(2, 0)
(0, 1)	(1, 1)	(2, 1)
(0, 2)	(1, 2)	(2, 2)

Table

x	0	1	2	3	4
P(x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

Make histogram

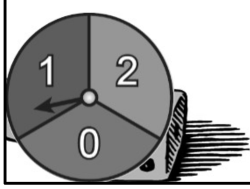
#1: Sample Space: 1, 1, 1, 1, 1, 2, 2, 2, 3, 3

Table

X	1	2	3
P(x)	0.5	0.3	0.2

8.7 Binomial Distributions

- Use the probability distribution in the previous slide to answer each question.
- What is the most likely sum when spinning the three-section spinner twice?
- What is the probability that the sum of the two spins is odd?



- Try 453#5

2

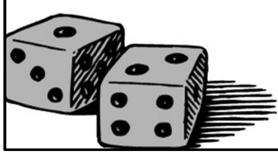
$\frac{2}{9} + \frac{2}{9} = \frac{4}{9} = 0.444$

#5a: 2 most likely; b: 5/8

8.7 Binomial Distributions

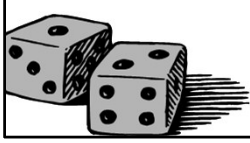
- Binomial Distributions
  - Two outcomes: Success or failure
  - Independent trials (*n*)
  - Probability for success is the same for each trial (*p*)

- $P(k\ successes) = {}_nC_k p^k (1-p)^{n-k}$



8.7 Binomial Distributions

- Calculate the probability of flipping a coin 20 times and getting 3 heads.
- Try 453#7
  only 1 head



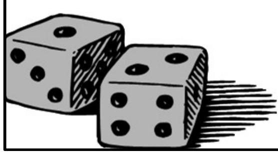
$$P(3) = {}_{20}C_3(0.5)^3(1-0.5)^{20-3} = 0.00109$$

#7

$$P(1) = {}_{20}\mathrm{C}_1(0.5)^1(1-0.5)^{20-1} = 0.0000191$$

8.7 Binomial Distributions

- At college, 53% of students receive financial aid. In a random group of 9 students, what is the probability that exactly 5 of them receive financial aid?

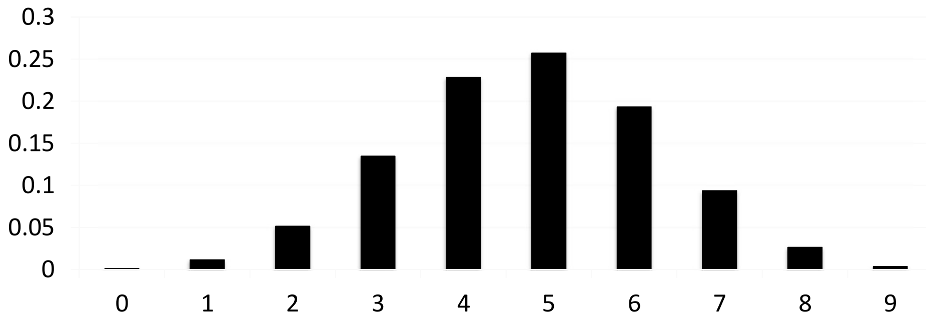


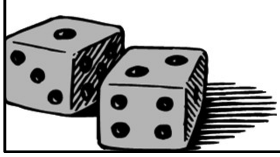
$$p = .53, n = 9, k = 5$$

$$P(5) = {}_9C_5(0.53)^5(1 - .53)^{9-5} = 0.257$$

8.7 Binomial Distributions

- Draw a histogram of binomial distribution of students in example 1 and find the probability of fewer than 3 students receiving financial aid.





Make a table in the calculator by entering

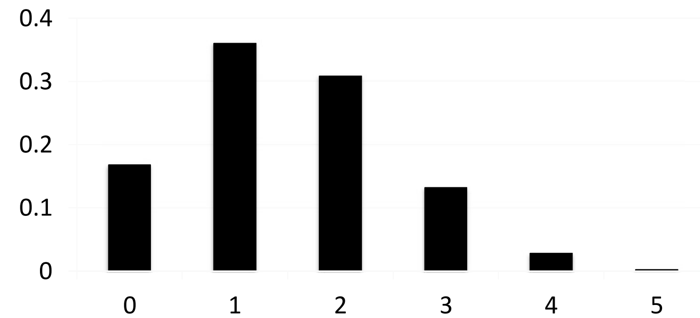
$$y = {}_9C_x \cdot 0.53^x \cdot (1-0.53)^{9-x}$$

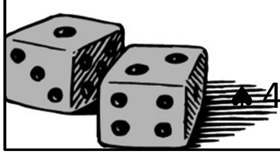
$$P(0) = {}_9C_0(0.53)^0(1-0.53)^{9-0} = 0.00112$$
$$P(1) = {}_9C_1(0.53)^1(1-0.53)^{9-1} = 0.01136$$
$$P(2) = .05123$$
$$P(3) = .13480$$
$$P(4) = .22801$$
$$P(5) = .25712$$
$$P(6) = .19330$$
$$P(7) = .09342$$
$$P(8) = .02634$$
$$P(9) = .00330$$

$$P(<3) = P(0) + P(1) + P(2) = .00112 + .01136 + .05123 = .06371$$

8.7 Binomial Distributions

- Try 453#11
  In your school, 30% of students plan to attend a movie night. You ask 5 randomly chosen students from your school whether they plan to attend the movie night.
  a. Draw a histogram
  b. Most likely
  c. Probability at most 2 attend





- 453 #1, 3, 5, 7, 9, 11, 12, 13, 15, 17, 19, 21, 23, 25, 27

a. Make a table in the calculator by entering

$$y = {}_5C_x \cdot 0.3^x \cdot (1-0.3)^{5-x}$$

$$P(0) = {}_5C_0(0.3)^0(1-0.3)^{5-0} = 0.16807$$
$$P(1) = {}_5C_1(0.3)^1(1-0.3)^{5-1} = 0.36015$$
$$P(2) = 0.3087$$
$$P(3) = 0.1323$$
$$P(4) = 0.02835$$
$$P(5) = 0.00243$$

b. 1 is most likely

c. $P(at\ most\ 2) = P(0) + P(1) + P(2) = 0.16807 + 0.36015 + 0.3087 = 0.83692$